BINOMIAL THEOREM

Binomial Expansion

$$(a+b)^n = \binom{n}{0}(a)^{n-0}(b)^0 + \binom{n}{1}(a)^{n-1}(b)^1 + \ldots$$

1. Second Term, b, include Sign
2. Apply Power to every value
3. Remember to add +…

Hence

Selective Expansion

Full Expansion

Appro-ximation

Finding Specific Term

$$T_{r+1} = \binom{n}{r}(a^{n-r})(b^r)$$

1. SPLIT every term
2. REARRANGE & COMBINE
3. Compare Powers to find r

Advance

Unknown Power

Decide Method

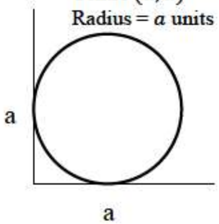
$$\binom{n}{0} = 1 \qquad \binom{n}{1} = n$$

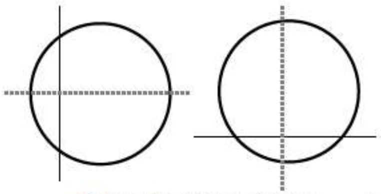
$$\binom{n}{2} = \frac{(n)(n-1)}{2 \times 1}$$

$$\binom{n}{3} = \frac{(n)(n-1)(n-2)}{3 \times 2 \times 1}$$

CIRCLES

Centre (a, a)
Radius = a units





(Vertical/Horizontal Chord)
Centre is Midpoint
of the 2 Intersections

Forming Equation

$$(x-a)^2 + (y-b)^2 = r^2$$

Centre (a, b) Radius: r

Finding Centre & Radius

$$x^2 + y^2 + 6x - 4y + 9 = 0$$

$$x^2 + 6x + y^2 - 4y = -9$$
$$(x+3)^2 + (y-2)^2 = -9 + 3^2 + 2^2$$
$$\therefore (x+3)^2 + (y-z)^2 = 2^2$$

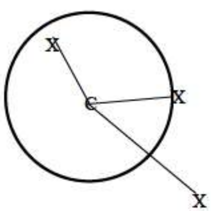
Hence, centre = $(-3, 2)$, Radius = 2 units

Follow
Complete The Square Rules <u>OR</u>
'Shortcut Formula'

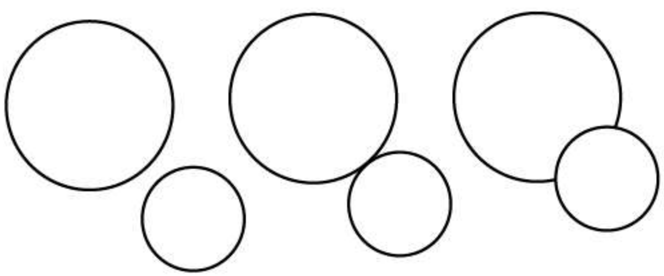
$$x^2 + 6x + y^2 - 4y = -9$$
$$x^2 + 2gx + y^2 + 2fy + c = 0$$

Centre $(-g, -f)$, Radius $\sqrt{g^2 + f^2 - c}$
Centre $(-3,2)$ Radius $\sqrt{9+4-9} = 2$

Position of Coordinates



1. Find length of line (Centre to Coordinate)
2. If $l > r$, outside circle
3. If $l = r$, on circle
4. If $l < r$, inside circle

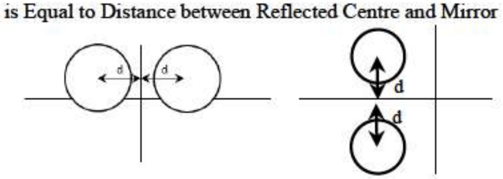


Reflection About Line

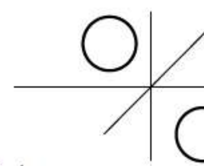
1. Always remember to sketch.
2. **Radius of the circle remains the same.**

Reflection about <u>Vertical</u> & <u>Horizontal</u> Lines

1. Remember that the Distance between Centre and Mirror is Equal to Distance between Reflected Centre and Mirror



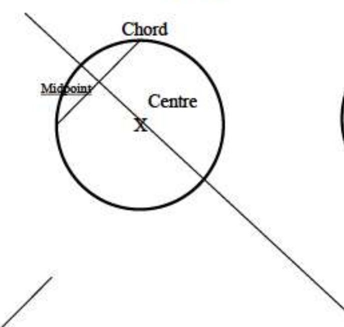
Reflection about <u>Diagonal Lines</u>

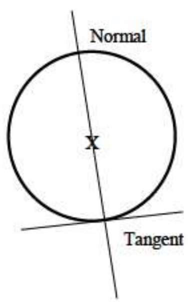


1. Find the Gradient of Diagonal Line (Mirror)
2. Find the Gradient of Perpendicular Line to the Mirror
3. Form Equation of Perpendicular Line passing through Centre
4. Simultaneous Equation between Equation of Perpendicular Line & Mirror
5. Use Midpoint Theory to find the Reflected Centre
6. Form the Reflected Circle Equation

Tangent, Chords & Perpendicular Bisector

Perpendicular Bisector of Chord
passes through Centre*

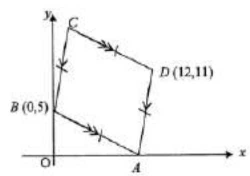


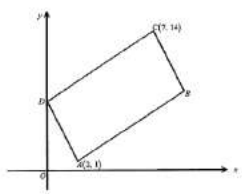


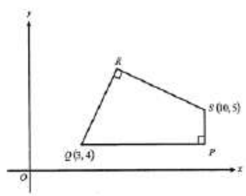
Perpendicular Bisector

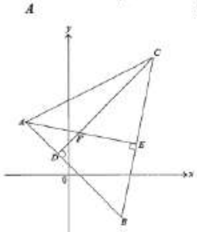
1. Gradient of line: $m = \frac{y_1 - y_2}{x_1 - x_2}$
2. Equation of line: $y - y_1 = m(x - x_1)$
3. Length of line: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
4. Midpoint Theory: $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$
5. Gradient of Perpendicular Bisector $= -\frac{1}{\text{Gradient of Tangent}}$
6. Perpendicular Bisector

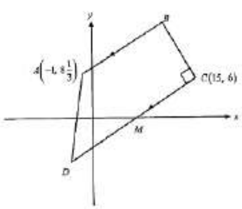
   -Find Midpoint of Line
   -Find Gradient of Line
   -Find Gradient of Perpendicular Bisector
   -Find Equation of Perpendicular Bisector (Midpoint & Grad)





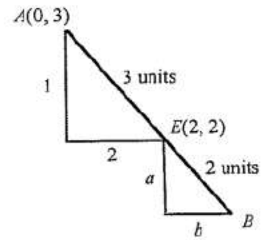






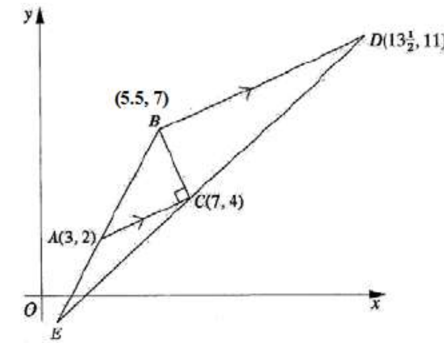
Collinear (Similar Triangle)

1. Ratio of Triangles
2. Method 1 vs Method 2



Area (Shoe-Lace Method)

1, Pick 1 coordinate
2. Anti Clockwise
3. Repeat First Coordinate



Area of $ABDC$

$$= \frac{1}{2}\begin{vmatrix} 3 & 7 & 13\frac{1}{2} & 5\frac{1}{2} & 3 \\ 2 & 4 & 11 & 7 & 2 \end{vmatrix}$$

= 22.5units

DIFFERENTIATION

Chain Rule

$$\frac{d}{dx}(ax+b)^n = (n)(ax+b)^{n-1}(a)$$

Product Rule

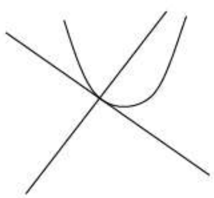
$$\frac{d}{dx}f(x)g(x) = f'(x)g(x) + g'(x)f(x)$$

Quotient Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$$

Equation of Tangent & Normal

1. Gradient of Tangent
2. Gradient of Normal
3. Forming Equations



Increasing & Decreasing Functions

1. Finding Range
-Quadratic Inequalities
-Reverse Quadratic Inequalities
-Explanation

2. Proving Questions
-Prove by Deduction
-Prove by Completing The Square

(Connected) Rate of Change

1. Basic Questions

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

Decreasing Rate
*Put Negative

2. Advance Questions

$\frac{dx}{dt} = k \times \frac{dy}{dt}$

"Double Split"

Mensuration

*Similar Triangles
*Pythagoras Theorem
*TOA CAH SOH

Maxima & Minima

1. First Derivative Test (Box)
2. Second Derivative Test

Coordinate Geometry

Mensuration

*Similar Triangles
*Pythagoras Theorem
*TOA CAH SOH

Trigonometry

Differentiate $sinx, cosx, tanx$ only
Use Trigo Identities for the rest
Process: Power Trigo Bracket

Exponential

$$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$$

Recall Law of Indices

Logarithm

$$\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}$$

Recall Law of Logarithm

EXPONENTIAL

Simplifying

Laws of Indices

Basic Rules	**Negative Powers**	**Fractional Powers**	**Zero Powers**
$y^a \times y^b = y^{a+b}$ $y^a \div y^b = y^{a-b}$ $(y^n)^m = y^{nm}$	$y^{-1} = \frac{1}{y}$ $\left(\frac{x}{y}\right)^{-1} = \left(\frac{y}{x}\right)$	$y^{\frac{1}{2}} = \sqrt{y}$ $y^{\frac{1}{3}} = \sqrt[3]{y}$	$y^0 = 1$

Word Problems

1. Standard Exponential Solving Questions

Advance Questions
1. Take note of Inequality Question
2. ROUND UP/DOWN
3. Infinity/Long Run Questions

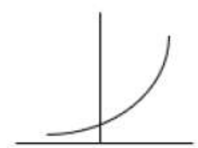
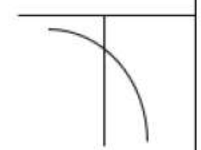
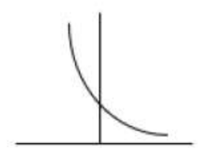
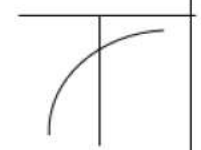
Solving

Compare Power
1. Change Base
2. Combine to Single Base
3. Compare **Powers**

Add Ln
1. Change Base
2. Combine Base
   OR
   Combine Powers (Adv.)
3. Add Ln and Solve

Substitution
1. Change Base
2. Split Powers
3. EXACT SAME Exponential, apply Substitution

Graphs

	$y = a^x$	$y = -a^x$
$a > 1$ $\log_a x = y$ $x = a^y$		
$0 < a < 1$		

INTEGRATION

Indefinite Integral

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)(a)} + C$$

Definite Integral

$$\int_b^a (ax+b)^n dx = \left[\frac{(ax+b)^{n+1}}{(n+1)(a)}\right]_b^a$$

Trigonometry

$$\int p\sin(qx+r)\,dx = \frac{-p\cos(qx+r)}{q} + C$$

$$\int p\cos(qx+r)\,dx = \frac{p\sin(qx+r)}{q} + C$$

$$\int p\sec^2(qx+r)\,dx = \frac{p\tan(qx+r)}{q} + C$$

In our syllabus,
we only learn to integrate
$\sin x$, $\cos x$, $\sec^2 x$.
Apply identities if other trigo are tested.

Exponential

$$\int e^{f(x)} dx = \frac{e^{f(x)}}{f'(x)} + C$$

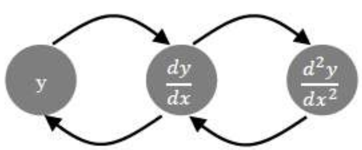
Recall Law of Indices

Logarithm

$$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + C$$

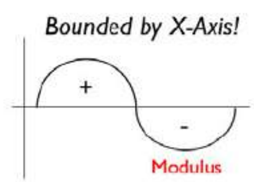
Recall Partial Fractions
If the Denominator is not Linear,
Apply Algebra Integration

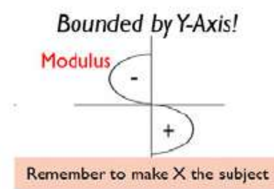
Curves



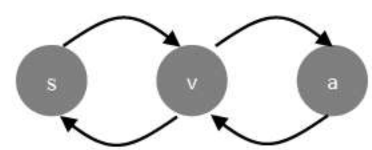
Rate of Change

Area (Region)





Kinematics



1. Find displacement, velocity, acceleration
2. Find time during Instantaneous Rest
3. Find minimum/maximum velocity
4. Total Distance (First 5s vs. Fifth Second)
5. Average Speed
6. 2 Collisions (Same distance travelled)

LINEAR LAW

Paper 1

1. Equation of Line: $y - y_1 = m(x - x_1)$
We can replace the x and y base on the AXIS.
LINEARISING the equation.

2. Process of Linearizing Non-Linear Functions
Remember the generic formula:

$$y = mx + c$$

Gradient and y intercept MUST be a CONSTANT.

Paper 2

Are you in the Linear World or Non-Linear World (Curve)?

Remember that we sketch NEW y axis against NEW x axis.

1. Linearise Equation
2. Find New Coordinates
3. Draw your Line
4. Find Gradient & Y Intercept

LOGARITHM

Simplifying

$\log_a y$ is read as logarithm of y to base a

$\log_a a = 1$ $\qquad \log_{10} a = \lg a$

$\log_a 1 = 0$ $\qquad \log_e e = \ln e = 1$

Laws of Logarithm

Product Law: $\log_a x + \log_a y = \log_a(xy)$

Quotient Law: $\log_a x - \log_a y = \log_a \frac{x}{y}$

Power Law: $r \times \log_a x = \log_a x^r$

Changing Base: $\log_a b = \frac{\log_c b}{\log_c a}$

New Base

Changing Log to Index Form: $\log_a x = y \rightarrow x = a^y$

Solving

1 Log

Log to Index

1. $\log_a y = x$
2. $y = a^x$

Multiple Logs

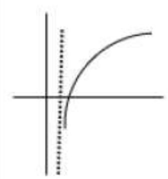
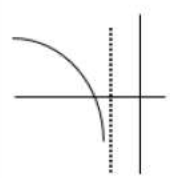
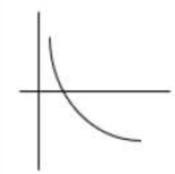
CCC

1. **Change to same base**
2. **Combine to single Log**
3. **Cancel Log on both side**

Substitution

1. Change to same base
2. EXACT SAME Log, apply Substitution

Graphs

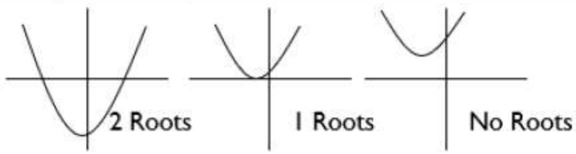
$y = \log_a x$, where $a > 1$	$y = \log_a x$, where $a > 1$	$y = \log_a x$, where $0 < a < 1$
 $y = \ln x$	 $y = -\ln x$	 $y = \log_{\frac{1}{3}} x$

NATURE OF ROOTS

Finding Ranges

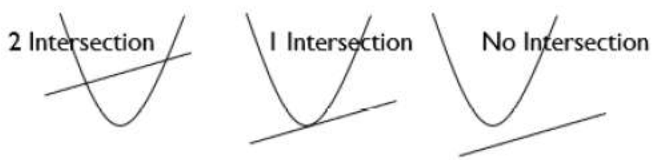
Determinants (Curve & Axis)

$b^2 - 4ac < 0$	**No Roots**	No Real Roots or Imaginary Roots or Graph is always positive (Completely Above x-axis) or Graph is always negative (Completely below x-axis)
$b^2 - 4ac = 0$	**1 Roots**	Real & Equal or Real & Repeated Roots
$b^2 - 4ac > 0$	**2 Roots**	Real & Distinct Roots or Different Roots
$b^2 - 4ac \geq 0$	**1 / 2 Roots**	Graph has real roots or Graph Intersects the x axis



Determinants (Curve & Line)

$b^2 - 4ac < 0$	**No Intersect**	Line Does Not Intersect The Curve
$b^2 - 4ac = 0$	**1 Intersect**	Line is tangent to Curve or Intersects Curve at one Point
$b^2 - 4ac > 0$	**2 Intersect**	Line Intersects curve at 2 points
$b^2 - 4ac \geq 0$	**1 / 2 Intersect**	Line Intersects / Meets Curve (May mean 1 or 2 points, consider both)



When to Reject?

We reject ranges on these scenarios:

1) Graph of $ax^2 + bx + c$, a cannot be 0.
2) Graph is always Positive, coefficient of x^2 cannot be Negative.
3) Graph is always Negative, coefficient of x^2 cannot be Positive.

Proving & Showing

Deduction

Apply this method when Conditions are given.

This allows you to break the equation into smaller pieces and explain step by step.

This is why it's called Proving through Deduction.

Completing The Square

Apply this method when you see a **Quadratic Equation.**

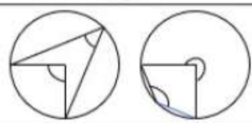
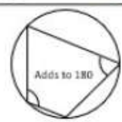
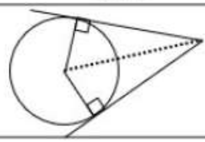
We are unable to prove whether $k^2 - 20k + 111$ is always Positive or Negative.

Through Completing The Square, we transform the equation into $(k - 10)^2 + 12$.

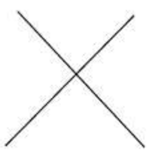
Now, we can easily explain ☺

PLANE GEOMETRY

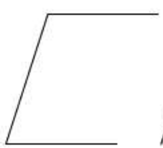
Circle Properties

Angles in the same segment are equal.	
Angle in a semicircle is a right angle, 90.	
Angle at the centre is twice the angles at the circumference.	
Angles in opposite segments of a circle are supplementary. (opposite angles in a cyclic quadrilateral are supplementary.)	
A tangent to a circle is perpendicular to the radius at the point of contact. Tangent from external point are equal.	

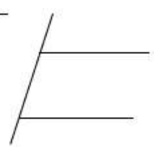
Angle Properties



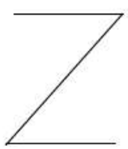
Vertically Opposite Angles



Interior Angles



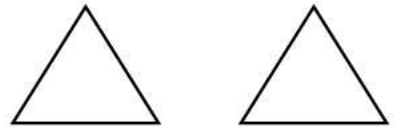
Corresponding Angles



Alternate Angles

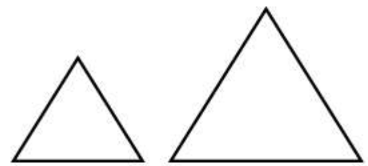
Congruence and Similarity

Congruent Triangles



You can prove by SSS, SAS, AAS, RHS
You **CANNOT** prove using AAA or ASS!

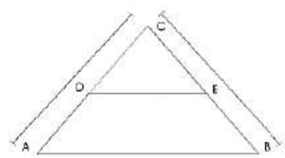
Similar Triangles



All 3 Corresponding Angles are same (AA)

All 3 Corresponding Sides have the same **RATIO**.

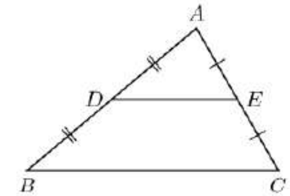
2 of the Corresponding Sides have the same **RATIO** & included Angles are the same.



Triangle CDE is similar to CAB.

$\frac{DC}{AC} = \frac{EC}{BC}$ Diagram is confusing, Look the name of triangle

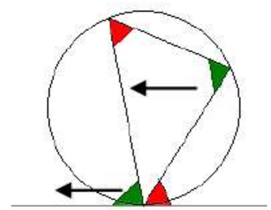
Midpoint Theorem

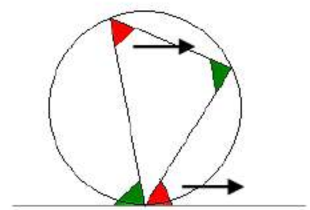


D is a midpoint of AB
E is a midpoint of AC
DE is parallel to BC

This means that ADE is similar to ABC.
Therefore, $DE = \frac{1}{2}BC$

Alternate Segment Theorem





POLYNOMIALS & PARTIAL FRACTION

Finding Unknown Values

Substitution Method
Compare Coefficients

Remainder/Factor Theorem

By RT, $f(x) = R$
By FT, $f(x) = 0$

Forming Original Equation*

1. Check Degree
2. Check Coefficient
3. Formation of Equation

Hence

1. Nature of Roots
2. Surds
3. Replacement Qn

Solving Cubic Equation

1. Mode 3,4 (Casio Calc)
2. Factor Theorem
3. Long Division
4. Solve

Factorising Cubic Equation

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

1. Check Improper vs Proper Fraction
2. If Improper, do Long Division
3. Fully Factorise Denominator

Case 1: Linear

Case 2: Square

Case 3: Quadratic

Case	Fraction $\frac{N(x)}{D(x)}$	Form of denominator, D(x)	Partial Fraction Form (where A, B and C are unknown constants)
1	$\frac{N(x)}{(ax+b)(cx+d)}$	Linear Factors	$\frac{A}{ax+b} + \frac{B}{cx+d}$
2	$\frac{N(x)}{(ax+b)^2}$	Repeated Linear Factors	$\frac{A}{ax+b} + \frac{B}{(ax+b)^2}$
	$\frac{N(x)}{(ax+b)(cx+d)^2}$	Linear and Repeated Linear Factors	$\frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}$
3	$\frac{N(x)}{(ax+b)(x^2+c^2)}$	Linear and Quadratic (which cannot be factorised) Factors	$\frac{A}{ax+b} + \frac{Bx+C}{x^2+c^2}$

$$\frac{1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$\frac{1}{(x+2)(x^2+3)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+3}$$

$$\frac{1}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$$

Completing The Square

$$x^2 - 3x + 7$$

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right)^2 + 7$$

$$(x - \frac{3}{2})^2 + \frac{19}{4}$$

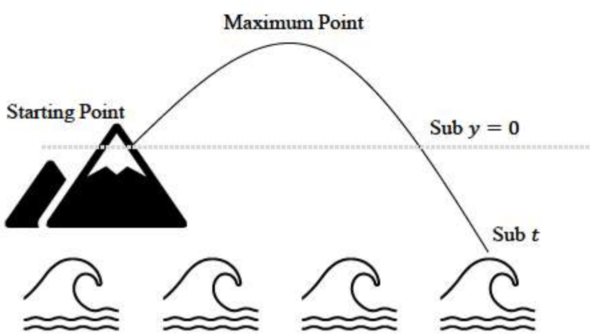
Coefficient of x^2 must be: Positive and One

Factorise if it is not e.g.
$-2x^2 + 4x + 8$
$= -2(x^2 - 2x - 4)$
Then you conduct C.T.S, on the expression in Bracket.

When x^2 *is not* 1

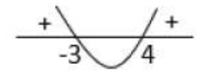
$$-2x^2 + 4x + 8$$
$$= -2(x^2 - 2x - 4)$$
$$= -2[(x^2 - 2x + (\frac{-2}{2})^2 - (\frac{-2}{2})^2 - 4]$$
$$= -2[(x - 1)^2 - 5]$$
$$= -2(x - 1)^2 + 10$$

Application

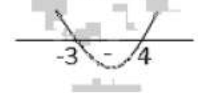


Quadratic Inequalities

$(x + 3)(x - 4) > 0$
$x < -3$ *or* $x > 4$



$(x + 3)(x - 4) < 0$
$-3 < x < 4$



Reverse Quadratic Inequality

Step 1: Flush everything to the left and rearrange according to $ax^2 + bx + c$
Step 2: Simplify and rearrange according to $ax^2 + bx + c$
Step 3: Solve your quadratic inequalities

Note: Always ensure x^2 is Positive,
If it's negative, divide and FLIP Your INEQUALITY SIGN.

Reverse Inequalities

Step 1: Given that $x < -3$ or $x > 2$
Step 2: $(x + 3)(x - 2) > 0$ (Reverse and Form Back Original)
Step 3: $x^2 + x - 6 > 0$ (Expand)

Simplifying

Multiplication:

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
$a \times \sqrt{b} = a\sqrt{b}$
$c\sqrt{a} \times d\sqrt{b} = cd\sqrt{ab}$
$\sqrt{a} \times \sqrt{a} = a$
$b\sqrt{a} \times b\sqrt{a} = b^2 a$

Number × Number,
Surd × Surd

Division:

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Addition & Subtraction:

$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$
$4\sqrt{2} - \sqrt{2} = 3\sqrt{2}$

Similar Terms can be added or subtracted

Key to Solving Surds:
Simplify all Surds to their simplest forms

$\sqrt{50} = 5\sqrt{2}$
$\sqrt{27} = 3\sqrt{3}$

Rationalisation

$$\frac{2}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$\frac{2}{\sqrt{5} - 3} \times \frac{\sqrt{5} + 3}{\sqrt{5} + 3}$$

$$\frac{2}{\sqrt{5} + 3} \times \frac{\sqrt{5} - 3}{\sqrt{5} - 3}$$

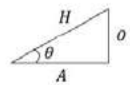
Train your speed in Surds Expansion.
It is back to Special Products.

$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)(a + b) = (a^2 - b^2)$

TRIGONOMETRY

Simplifying

1. Trigonometric Special Angles
2. Basic Angles
3. Trigonometric Identities
4. Addition Formula
5. Double Angle Formula
6. Half Angle Formula



Pythegoras Theorem, $H^2 = A^2 + O^2$

$tan\theta = \frac{O}{A}$ $cos\theta = \frac{A}{H}$ $sin\theta = \frac{O}{H}$

Angle Conversion
(Degree & Radian)
$180° = \pi\ rad$

Special Angle

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
[illegible]	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
[illegible]	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Negative Angles

$\sin(-\theta) = -sin\theta$
$\cos(-\theta) = cos\theta$
$\tan(-\theta) = -tan\theta$

Trigo Functions

$sec\theta = \frac{1}{tan\theta}$
$cosec\theta = \frac{1}{sin\theta}$
$cot\theta = \frac{1}{tan\theta}$

Addition

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Double Angle

$\sin(2\theta) = 2sin\theta cos\theta$
$\cos(2\theta) = 2cos^2\theta - 1$
$= 1 - 2sin^2\theta$
$= cos^2\theta - sin^2\theta$
$\tan(2\theta) = \frac{2tan\theta}{1 - tan^2\theta}$

Half Angle

$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - cos\theta}{2}}$

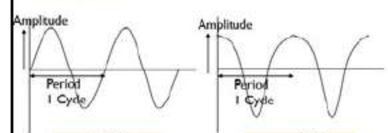
$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + cos\theta}{2}}$

The sign depends entirely on which quadrant $\frac{\theta}{2}$ lies in

Graphs

1. Basic Graph Shapes (sin, cos, tan)
2. Obtaining Amplitude, Period, Shifting
3. Application to Real World Context

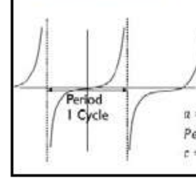
Graphs



$y = a\sin(b\theta) + c$ $y = a\cos(b\theta) + c$

$a = Amplitude\ (Height\ from\ Origin)$
$Period = \frac{360}{b}\ (Thinner\ or\ Fatter)$
$c = Shift\ Up\ or\ Down$

$y = a\tan(b\theta) + c$



$a = Amplitude(Infinity)$
$Period = \frac{180}{b}\ (Thinner\ or\ Fatter)$
$c = Shift\ Up\ or\ Down$

Quadrants

Step 1: Identify Quadrant
Step 2: Draw your Triangle
Step 3: Label the Sides of the Triangle
(Please be careful of the Signs)
Step 4: Find all the sides (Pythagoras)
Step 5: Solve

Solving

1. Simplifying
2. Basic Angle
   -Ensure it is Positive
   -Check Radian or Degree
3. Quadrant (ASTC)
4. Domain (Change Domain if required)
5. Solve

R Formula

1. Find Right Angle Triangle
2. Find more Theta, θ
3. Never CUT Theta, CUT 90°
4. Max/Min Value & it's θ
5. Solving

R Formula

$a\sin\theta \pm b\cos\theta = R\sin(\theta \pm \alpha)$
$a\cos\theta \pm b\sin\theta = R\cos(\theta \mp \alpha)$

$R = \sqrt{a^2 + b^2}$, $\alpha = tan^{-1}(\frac{b}{a})$

Step 1: Prove Equation
Step 2: Apply R Formula
Step 3: Application Question
Max/Min Value, Find Value